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$s_B: s(B) \rightarrow B$ が fibration であることの確認

$$(K, Z) \xrightarrow{(v, g)} (I, Y) \xrightarrow{(u, \pi')} (J, Y)$$

$(u \circ v, f)$

$$K \xrightarrow{v} I \xrightarrow{u} J$$

$$(u, \pi') \circ (v, g) = (u \circ v, f)$$

$$\Leftrightarrow g = f$$

$\therefore (v, f)$ は条件を満たす一意な射

$$B \xrightarrow[\cong]{1^*} B//1$$

$$B(X, Y) \cong B(1 \times X, Y) = B//1(X, Y)$$

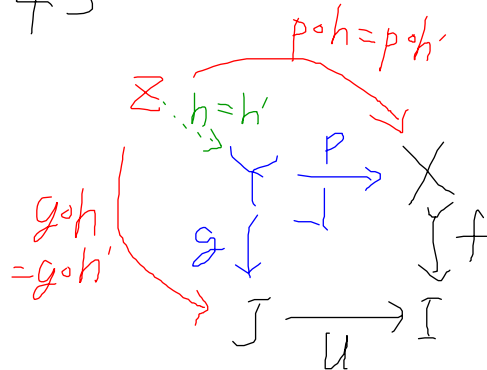
$$Ob(B) = Ob(B//1)$$

$$B \xrightarrow[\cong]{1^*} B/1$$

$$B(X, Y) \cong B/1(X, Y)$$

$$Ob(B) \cong Ob(B/1)$$

P. 43



A pullback of a mono along ^{an} arbitrary map is mono again.

$$h, h' : Z \rightarrow Y$$

$$g \circ h = g \circ h' \text{ となる}$$

$$f \circ p \circ h = u \circ g \circ h$$

$$= u \circ g \circ h'$$

$$= f \circ p \circ h'$$

f が mono なのので

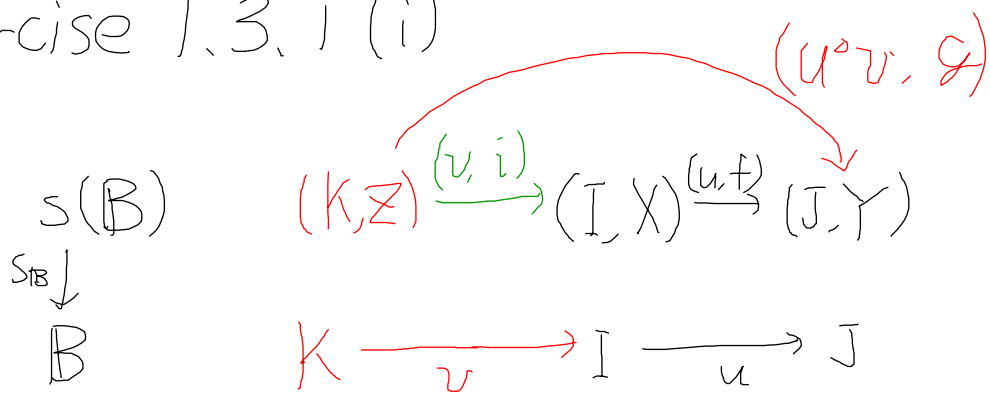
$$p \circ h = p \circ h'$$

よって h, h' は左上の図式を
可換にする一意な射

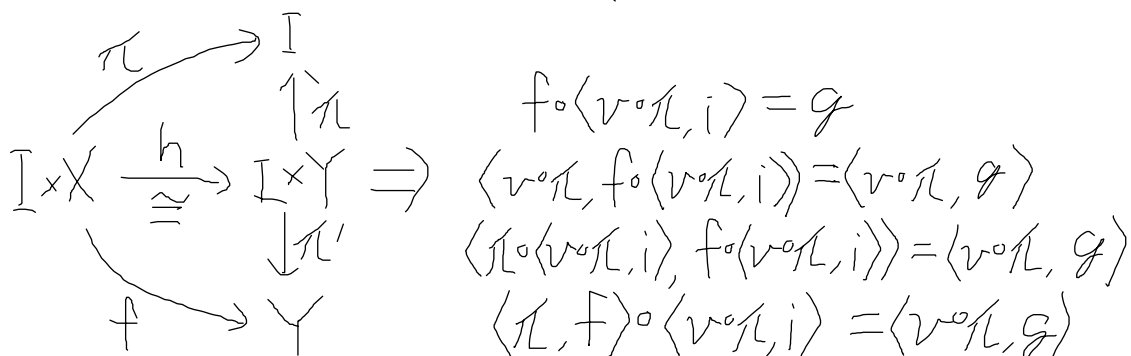
$$\therefore h = h'$$

$\therefore g$ は mono

Exercise 1.3.1 (i)



$$(u, f) \circ (v, i) = (u \circ v, f \circ \langle v \circ \pi, i \rangle)$$



$$f \circ \langle v \circ \pi, i \rangle = g$$

$$\langle v \circ \pi, f \circ \langle v \circ \pi, i \rangle \rangle = \langle v \circ \pi, g \rangle$$

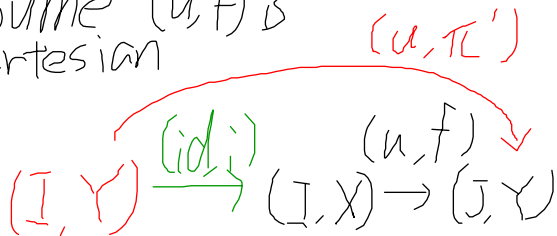
$$\langle \pi \circ \langle v \circ \pi, i \rangle, f \circ \langle v \circ \pi, i \rangle \rangle = \langle v \circ \pi, g \rangle$$

$$\langle \pi, f \rangle \circ \langle v \circ \pi, i \rangle = \langle v \circ \pi, g \rangle$$

$$\langle v \circ \pi, i \rangle = h^{-1} \circ \langle v \circ \pi, g \rangle$$

$$i = \pi' \circ h^{-1} \circ \langle v \circ \pi, g \rangle$$

assume (u, f) is cartesian



$\hookrightarrow \exists (u, f)$ is cartesian

$$I \xrightarrow{id} I \xrightarrow{u} J$$

$i: I \times Y \rightarrow X$ such that $f \circ \langle \pi, i \rangle = \pi'$

$$\begin{aligned} & \langle \pi, f \rangle \circ \langle \pi, i \rangle \\ &= \langle \pi \circ \langle \pi, i \rangle, f \circ \langle \pi, i \rangle \rangle \\ &= \langle \pi, \pi' \rangle = id_{I \times Y} \end{aligned}$$

$$\begin{aligned} & \langle \pi, i \rangle \circ \langle \pi, f \rangle \\ &= \langle \pi \circ \langle \pi, f \rangle, i \circ \langle \pi, f \rangle \rangle \\ &= \langle \pi, \pi' \rangle = id_{I \times X} \end{aligned}$$

省略

Exercise 1.3.1 (ii)

$$\begin{array}{ccc}
 s(B) & & B \\
 (\underline{I}, X) \mapsto I^*(X) = & \begin{array}{c} I \times X \\ \downarrow \pi \\ I \end{array} & \\
 (u, f) \downarrow & & \\
 (\underline{J}, Y) \mapsto J^*(Y) = & \begin{array}{c} J \times Y \\ \downarrow \pi \\ J \end{array} & \\
 & & \left. \begin{array}{c} \\ \\ \end{array} \right\} u^* f
 \end{array}$$

$$\begin{array}{ccc}
 I \times X & \xrightarrow{\langle u \circ \pi, f \rangle} & J \times Y \\
 \pi \downarrow \wr & & \downarrow \pi \\
 I & \xrightarrow{u} & J
 \end{array}$$

$u: I \rightarrow J$
 $f: I \times X \rightarrow Y$

idの保存

$$\begin{array}{ccc}
 (\underline{I}, X) \xrightarrow{\text{id}_{(\underline{I}, X)} = \langle \text{id}_I, \pi \rangle} (\underline{I}, X) & \mapsto & \begin{array}{ccc} I \times X & \xrightarrow{\langle \pi, \pi \rangle = \text{id}} & I \times X \\ \pi \downarrow \wr & \wr \text{id}_I^* \pi' & \downarrow \pi \\ I & \xrightarrow{\text{id}} & I \end{array}
 \end{array}$$

compositionの保存

$$(\underline{I}, X) \xrightarrow{\langle u, f \rangle} (\underline{J}, Y) \xrightarrow{\langle v, g \rangle} (\underline{K}, Z) \mapsto \begin{array}{ccccc} I \times X & \xrightarrow{\langle u \circ \pi, f \rangle} & J \times Y & \xrightarrow{\langle v \circ \pi, g \rangle} & K \times Z \\ \pi \downarrow \wr & & \pi \downarrow \wr & & \pi \downarrow \wr \\ I & \xrightarrow{u} & J & \xrightarrow{v} & K \end{array}$$

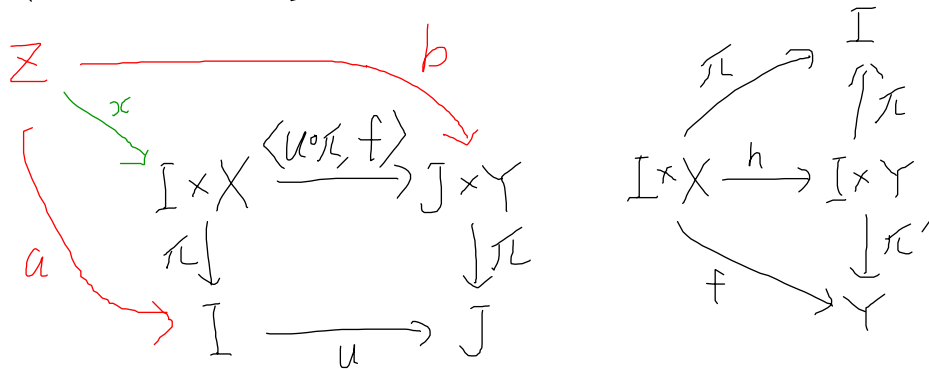
$$\langle v \circ u, g \circ \langle u \circ \pi, f \rangle \rangle$$

$$\begin{array}{ccc}
 \langle v \circ \pi, g \rangle \circ \langle u \circ \pi, f \rangle & & \begin{array}{ccc} I \times X & \xrightarrow{\langle v \circ u \circ \pi, g \circ \langle u \circ \pi, f \rangle \rangle} & K \times Z \\ \pi \downarrow \wr & & \pi \downarrow \wr \\ I & \xrightarrow{v \circ u} & K \end{array} \\
 = \langle v \circ u \circ \pi, g \circ \langle u \circ \pi, f \rangle \rangle & &
 \end{array}$$

続く

Exercise 1.3.1 (ii) 系統

$(u, f) : (I, X) \rightarrow (J, Y)$ が cartesian である



$u \circ a = \pi \circ b$ を満たす a, b が与えられたとする

$$\begin{aligned}
 & \pi \circ x = a \wedge f \circ x = \pi' \circ b \\
 \Rightarrow & \langle u \circ \pi, f \rangle \circ x \\
 & = \langle u \circ \pi \circ x, f \circ x \rangle \\
 & = \langle u \circ a, \pi' \circ b \rangle \\
 & = \langle \pi \circ b, \pi' \circ b \rangle \\
 & = b
 \end{aligned}$$

$$\begin{aligned}
 & \left(\begin{array}{l} \pi \circ x = a \\ \langle u \circ \pi, f \rangle \circ x = b \end{array} \right) \\
 \Leftrightarrow & \left(\begin{array}{l} \pi \circ x = a \\ f \circ x = \pi' \circ b \end{array} \right) \\
 \Leftrightarrow & h \circ x = \langle \pi, f \rangle \circ x = \langle a, \pi' \circ b \rangle \\
 \Leftrightarrow & x = h^{-1} \circ \langle a, \pi' \circ b \rangle
 \end{aligned}$$

よって条件を満たす x が一意に存在

ゆえに u^*f は pullback square

Exercise 1.3.2 (i) B が finite product を持つ
とする

B/I が finite product を持つことを示す

$$\begin{aligned} B(I \times X, 1) \\ \cong B/I(X, 1) \end{aligned}$$

$$\begin{aligned} B/I(Z, X) \times B/I(Z, Y) \\ \cong B(I \times Z, X) \times B(I \times Z, Y) \\ \cong B(I \times Z, X \times Y) \\ \cong B/I(Z, X \times Y) \end{aligned}$$

$I^*: B \rightarrow B/I$ が finite product を保存することは
明らか

Exercise 1.3.2(ii) \mathcal{B} が finite product を持つとする

(a) \mathcal{B} is cartesian-closed

(b) each fibre \mathcal{B}/I is cartesian closed

(c) each functor $I^*: \mathcal{B} \rightarrow \mathcal{B}/I$ has a right adjoint $I \Rightarrow (-)$

(a) \Rightarrow (b)

terminal object (i) で示した

product (i) で示した

$$\begin{aligned} \text{exponential } \mathcal{B}/I(X \times Y, Z) &= \mathcal{B}(I \times X \times Y, Z) \\ &= \mathcal{B}(I \times X, Y \Rightarrow Z) \\ &= \mathcal{B}/I(X, Y \Rightarrow Z) \end{aligned}$$

(b) \Rightarrow (a)

仮定より $\mathcal{B} \models$ terminal object \times product はある

$$\begin{aligned} \text{exponential } \mathcal{B}(X \times Y, Z) &\cong \mathcal{B}/X(Y, Z) \cong \mathcal{B}/X(1, Y \Rightarrow Z) \\ &\cong \mathcal{B}(X \times 1, Y \Rightarrow Z) \cong \mathcal{B}(X, Y \Rightarrow Z) \end{aligned}$$

(a) \Rightarrow (c)

$$\mathcal{B}/I(I^*(X), Y) \cong \mathcal{B}/I(X, Y) \cong \mathcal{B}(I \times X, Y) \cong \mathcal{B}(X, I \Rightarrow Y)$$

(c) \Rightarrow (a)

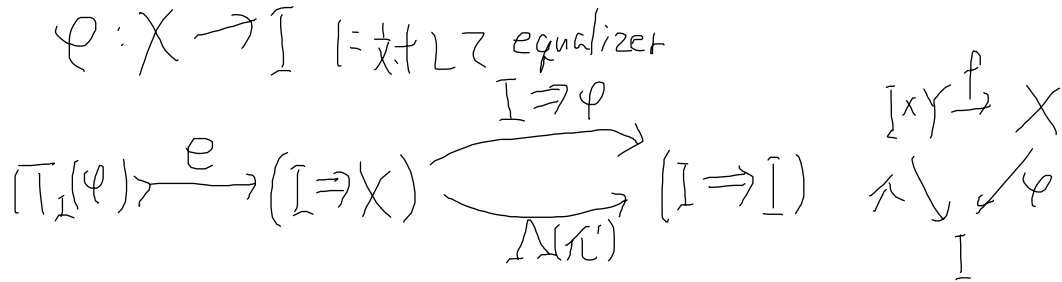
仮定より $\mathcal{B} \models$ terminal object \times product はある

$$\text{exponential } \mathcal{B}(X \times Y, Z) \cong \mathcal{B}/Y(Y^*(X), Z) \cong \mathcal{B}(X, Y \Rightarrow Z)$$

Exercise 1.3.3

\mathcal{B} has finite limit

各 $I^* : \mathcal{B} \rightarrow \mathcal{B}/I$ が right adjoint を持つ $\Leftrightarrow \mathcal{B}$ は CCC



の $\Pi_I(\varphi)$ が条件を満たすことを示す \rightarrow

各 $I^* : \mathcal{B} \rightarrow \mathcal{B}/I$ が right adjoint を持つ
 $\Rightarrow \mathcal{B}$ は CCC

$X \Rightarrow Y \triangleq \Pi_X X^* Y$ が条件を満たす

$$\begin{aligned} & \mathcal{B}(Z, \Pi_X X^* Y) \\ &= \mathcal{B}/X (X^* Z, X^* Y) \\ &= \{ f : X \times Z \rightarrow X \times Y \mid \pi \circ f = \pi \} \\ &= \{ g : X \times Z \rightarrow Y \} \\ &= \mathcal{B}(X \times Z, Y) \\ &= \mathcal{B}(Z \times X, Y) \end{aligned}$$

$$\begin{aligned} & \mathcal{B}/I (I^* \varphi) \\ &\cong \{ f : I^* Y \rightarrow X \mid \pi = \varphi \circ f \} \\ &\cong \{ g : Y \times I \rightarrow X \mid \pi' = \varphi \circ g \} \\ &\cong \{ h : Y \rightarrow [I \Rightarrow X] \mid \Delta(\pi') \circ h = (I \Rightarrow \varphi) \circ h \} \\ &\cong \{ h : Y \rightarrow [I \Rightarrow X] \mid \Delta(\pi') \circ h = (I \Rightarrow \varphi) \circ h \} \\ &\cong \mathcal{B}(Y, \Pi_I \varphi) \end{aligned}$$

Exercise 1.3.4 (i)

comonad $(I \times (-), \epsilon, \delta)$

with $\epsilon_x \triangleq \pi' : I \times X \rightarrow X$

$\delta_x \triangleq \langle \pi, \text{id}_{I \times X} \rangle : I \times X \rightarrow I \times I \times X$

$$\begin{array}{ccc}
 I \times I \times I \times X & \xleftarrow{I \times \delta_x} & I \times I \times X \\
 \delta_{I \times X} \uparrow & & \uparrow \delta_x \\
 I \times I \times X & \xleftarrow{\delta_x} & I \times X
 \end{array}$$

$$\begin{array}{ccc}
 I \times X & \xleftarrow{I \times \epsilon_x} & I \times I \times X & \xrightarrow{\epsilon_{I \times X}} & I \times X \\
 \uparrow & & \uparrow \delta_x & & \uparrow \\
 I \times X & & I \times X & & I \times X
 \end{array}$$

2つの図式は
明らかに可換

Exercis 1.3.4 (ii)

comonad $(I^x(-): \mathbb{B} \rightarrow \mathbb{B}, \varepsilon, \delta)$ の coKleisli category

Objects $X, Y \in \text{Ob}(\mathbb{B})$

Morphisms $\text{hom}(X, Y) \rightarrow \mathbb{B}(I^x X \rightarrow Y)$

$$\begin{aligned} \text{id}_X &\in \text{hom}(X, X) \\ &= \varepsilon_X \in \mathbb{B}(I^x X, X) \\ &= \pi' \end{aligned}$$

$$\begin{aligned} f \circ g &= f \circ_{\mathbb{B}} (I^x g) \circ \delta \\ &= f \circ_{\mathbb{B}} (I^x g) \circ \langle \pi, \text{id} \rangle \\ &= f \circ_{\mathbb{B}} \langle \pi, g \rangle \end{aligned}$$

$\hookrightarrow \tau \mathbb{B} / I \cong \mathbb{B}$

comonad $(I^x(-), \varepsilon, \delta)$ の Eilenberg-Moore category \mathbb{B} / I

Objects $(X, \varphi: X \rightarrow I^x X)$

such that $\varepsilon \circ \varphi = \text{id}_X$

$$\Leftrightarrow \pi \circ \varphi: X \rightarrow I \quad (= \text{射 } \pi \text{ に対する } \pi \text{ の lift})$$

Morphisms $(X, \varphi) \xrightarrow{h} (Y, \psi) \quad \psi \circ h = (I^x h) \circ \varphi$

$$\begin{array}{ccc} X & \xrightarrow{h} & Y \\ \varphi \downarrow & & \downarrow I^x \\ I^x X & \xrightarrow{I^x h} & I^x Y \end{array}$$

$$\Leftrightarrow \pi \psi h = \pi \varphi$$

$\hookrightarrow \tau \mathbb{B} / I \cong \mathbb{B}$

Exercise 1.3.5

$$\begin{array}{ccc} \begin{pmatrix} Z \\ \downarrow \\ K \end{pmatrix} & \begin{array}{c} \xrightarrow{(g,v)} \\ \xrightarrow{(h,w)} \end{array} & \begin{pmatrix} X \\ \downarrow \\ I \end{pmatrix} \xrightarrow{(f,u)} \begin{pmatrix} Y \\ \downarrow \\ J \end{pmatrix} \end{array}$$

(f, u) is mono $\Leftrightarrow \exists \delta \subseteq \mathcal{C}$
 $(f, u) \circ (g, v) = (f, u) \circ (h, w)$
 $\Leftrightarrow (fg, uv) = (fh, uw)$
 $\Leftrightarrow fg = fh \wedge uv = uw$
 $\Leftrightarrow g = h \wedge v = w$
 $\Leftrightarrow (g, v) = (h, w) \quad \text{so } (f, u) \text{ is mono}$

(f, u) is mono $\Leftrightarrow \exists \delta \subseteq \mathcal{C}$

$$\begin{array}{ccccc} Z & \xrightarrow{g} & X & \xrightarrow{f} & Y \\ & \searrow h & \downarrow \varphi & & \downarrow \\ Z & \xrightarrow{\varphi \circ g} & I & \xrightarrow{u} & J \\ & \searrow \varphi \circ h & & & \end{array}$$

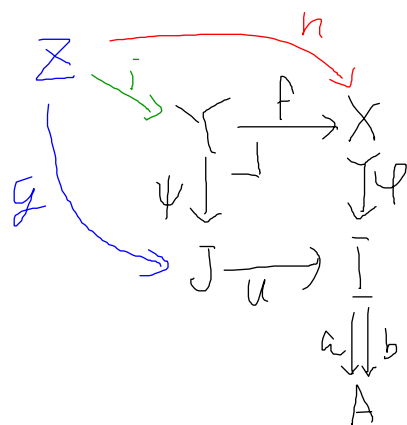
$fg = fh$
 $\Leftrightarrow (f, u) \circ (g, \varphi \circ g) = (f, u) \circ (h, \varphi \circ h)$
 $\Leftrightarrow (g, \varphi \circ g) = (h, \varphi \circ h)$
 $\Leftrightarrow g = h \quad \text{so } f \text{ is mono}$

Z is limit \Leftrightarrow
 定義

$$\begin{array}{ccccc} Z & \xrightarrow{g} & X & \xrightarrow{f} & Y \\ \downarrow & & \downarrow \varphi & & \downarrow \\ K & \xrightarrow{v} & I & \xrightarrow{u} & J \end{array}$$

$u \circ v = u \circ w$
 $\Leftrightarrow (f, u) \circ (g, v) = (f, u) \circ (g, w)$
 $\Leftrightarrow (g, v) = (g, w) \Leftrightarrow v = w$
 so u is mono

Exercise 1.3.6



問題文には書いて
いないけれど、 B が P_B を
持つことは仮定して
良いんだよね？

ϕ が "regular mono \mathcal{C} "

a, b の equalizer $\tau \in \mathcal{C}$ がある

$$a \circ u \circ g = b \circ u \circ g$$

\Rightarrow - ϕ が equalizer τ の \mathcal{C} -

$$\exists ! h: Z \rightarrow X, \phi \circ h = u \circ g$$

\Rightarrow - P_B -

$$\exists ! i: Z \rightarrow Y, \psi \circ i = g \wedge f \circ i = h$$

$\therefore \psi$ は $a \circ u, b \circ u$ の
equalizer τ であり
regular mono

Exercise 1.3.7 (i)

\mathcal{B} が finite limit を持つとする

\mathcal{B} が CCC $\Rightarrow \mathcal{B}^{\rightarrow}$ は CCC

terminal object
$$\begin{array}{ccc} X & \xrightarrow{!} & 1 \\ \varphi \downarrow & & \downarrow ! \\ I & \xrightarrow{!} & 1 \end{array}$$

product
$$\begin{pmatrix} X \\ \downarrow \varphi \\ I \end{pmatrix} \times \begin{pmatrix} Y \\ \downarrow \psi \\ J \end{pmatrix} = \begin{pmatrix} X \times Y \\ \downarrow \varphi \times \psi \\ I \times J \end{pmatrix}$$

$$\begin{aligned} \mathcal{B}^{\rightarrow}(r, \varphi \times \psi) &= \{ (f: Z \rightarrow X \times Y, u: K \rightarrow I \times J) \mid (\varphi \times \psi) \circ f = u \circ r \} \\ &= \{ (f_1: Z \rightarrow X, f_2: Z \rightarrow Y, u_1: K \rightarrow I, u_2: K \rightarrow J \mid \varphi \circ f_1 = u_1 \circ r, \psi \circ f_2 = u_2 \circ r) \} \\ &= \mathcal{B}^{\rightarrow}(r, \varphi) \times \mathcal{B}^{\rightarrow}(r, \psi) \end{aligned}$$

exponent
$$\begin{pmatrix} X \\ \downarrow \varphi \\ I \end{pmatrix} \Rightarrow \begin{pmatrix} Y \\ \downarrow \psi \\ J \end{pmatrix} \quad \begin{array}{ccc} U & \xrightarrow{\varphi} & (X \Rightarrow Y) \\ \downarrow \varphi \Rightarrow \psi & & \downarrow \varphi \Rightarrow \psi \\ (I \Rightarrow J) & \xrightarrow{\varphi \Rightarrow \psi} & (X \Rightarrow Y) \end{array}$$

$$\begin{aligned} \mathcal{B}^{\rightarrow}(r \times \varphi, \psi) &= \{ (f: Z \times X \rightarrow Y, u: K \times I \rightarrow J) \mid \psi \circ f = u \circ (r \times \varphi) \} \\ &\cong \{ (f': Z \rightarrow (X \Rightarrow Y), u': K \rightarrow (I \Rightarrow J)) \mid (X \Rightarrow Y) \circ f' = (\varphi \Rightarrow \psi) \circ u' \circ r \} \\ &\cong \{ (g: Z \rightarrow U, u: K \rightarrow (I \Rightarrow J)) \mid (\varphi \Rightarrow \psi) \circ g = u \circ r \} \\ &\cong \mathcal{B}^{\rightarrow}(r, \varphi \Rightarrow \psi) \end{aligned}$$

cod: $\mathcal{B}^{\rightarrow} \rightarrow \mathcal{B}$ が
CCC structure
を strictly 保つ
のは定義より
明らか

Exercise 1.3.7 (ii)

\mathcal{B} がfinite limitを持つ

\mathcal{B} がCCC \Rightarrow $\text{Sub}(\mathcal{B})$ はCCC

terminal object $\left(\begin{smallmatrix} 1 \\ \downarrow \\ 1 \end{smallmatrix}\right)$ はmono

product $\left(\begin{smallmatrix} X \\ \downarrow \varphi \\ I \end{smallmatrix}\right), \left(\begin{smallmatrix} Y \\ \downarrow \psi \\ J \end{smallmatrix}\right)$ に対して、 $(\varphi \times \psi) \circ \alpha = (\varphi \times \psi) \circ \beta$

$$\Leftrightarrow \varphi \circ \pi \circ \alpha = \varphi \circ \pi \circ \beta$$

$$\wedge \psi \circ \pi \circ \alpha = \psi \circ \pi \circ \beta$$

$\left(\begin{smallmatrix} X \times Y \\ \downarrow \varphi \times \psi \\ I \times J \end{smallmatrix}\right)$ は mono

$$\Leftrightarrow \pi \circ \alpha = \pi \circ \beta$$

$$\wedge \pi \circ \alpha = \pi \circ \beta$$

exponent $\left(\begin{smallmatrix} X \\ \downarrow \varphi \\ I \end{smallmatrix}\right), \left(\begin{smallmatrix} Y \\ \downarrow \psi \\ J \end{smallmatrix}\right)$ に対して、

$$\Leftrightarrow \alpha = \beta$$

$$\begin{array}{ccc} U & \longrightarrow & (X \Rightarrow Y) \\ \varphi \Rightarrow \psi \downarrow & & \downarrow X \Rightarrow Y \\ (I \Rightarrow J) & \xrightarrow{\varphi \Rightarrow J} & (X \Rightarrow J) \end{array}$$

$$(X \Rightarrow \psi) \circ f = (X \Rightarrow \psi) \circ g$$

$$\Leftrightarrow \psi \circ \text{ev} \circ (f \times \text{id})$$

$$= \psi \circ \text{ev} \circ (g \times \text{id})$$

$$\Leftrightarrow \text{ev} \circ (f \times \text{id}) = \text{ev} \circ (g \times \text{id})$$

$$\Leftrightarrow f = g$$

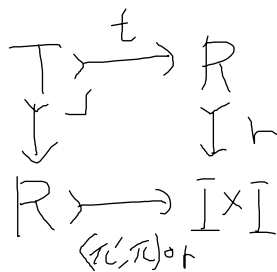
よって、 $(X \Rightarrow \psi)$ は mono

monoのpullback は mono

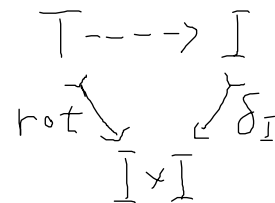
だから、 $\varphi \Rightarrow \psi \in \text{mono}$

Exercise 1.3.8

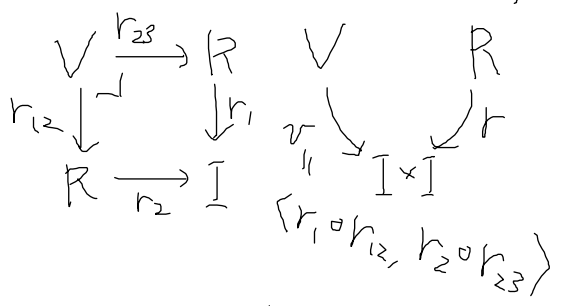
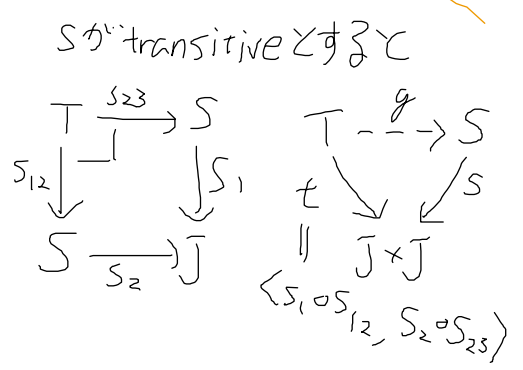
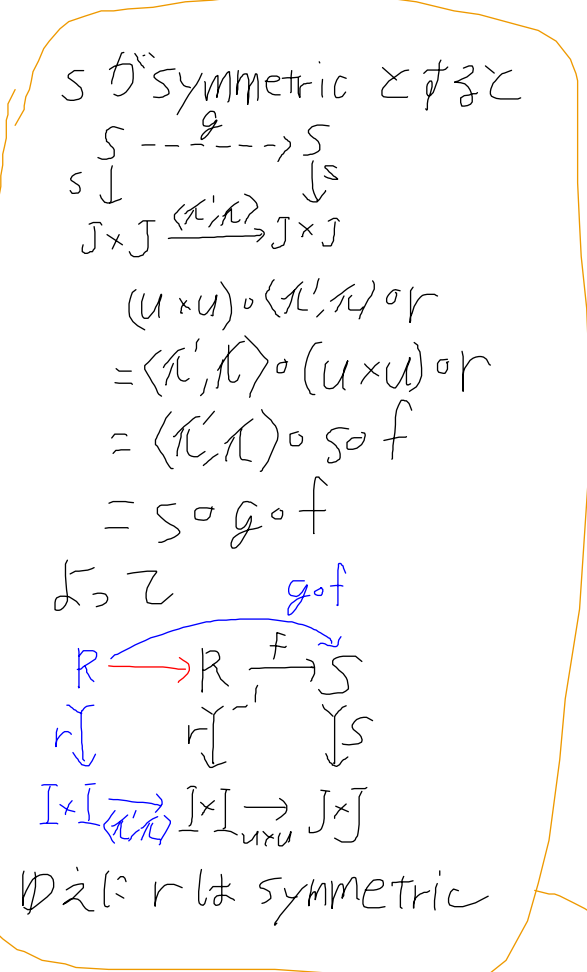
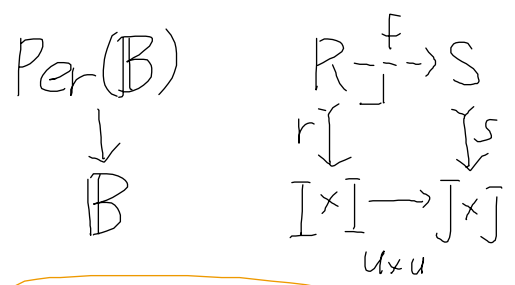
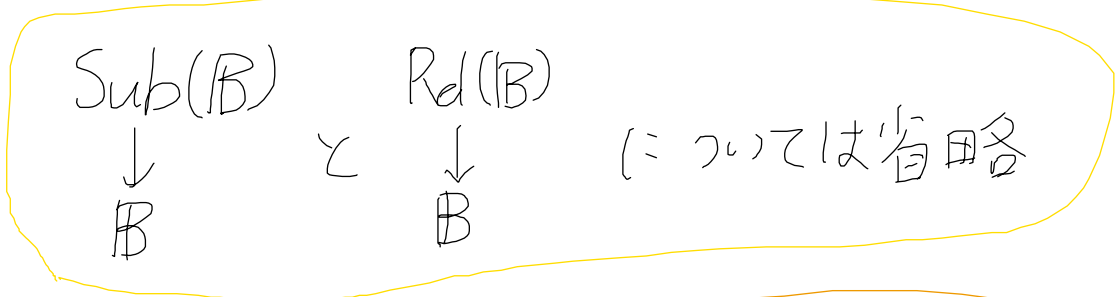
$r: R \rightarrow \bar{I} \times \bar{I}$ が anti-symmetric
 であることの定義



とおいて

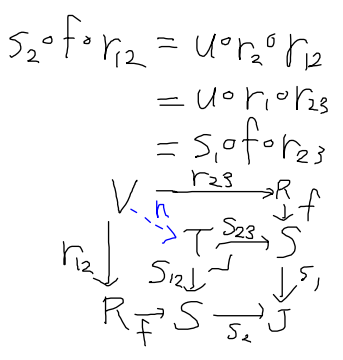


Exercise 1.3.9



$$\begin{aligned}
 (u \times u) \circ v &= \langle u \circ r_1 \circ r_{12}, u \circ r_2 \circ r_{23} \rangle \\
 &= \langle s_1 \circ f \circ r_{12}, s_2 \circ f \circ r_{23} \rangle \\
 &= \langle s_1 \circ s_{12} \circ h, s_2 \circ s_{23} \circ h \rangle \\
 &= t \circ h = s \circ g \circ h
 \end{aligned}$$

$\forall z: r$ は transitive



Exercise 1.3.9 続き

$$\begin{array}{ccc}
 R & \xrightarrow{f} & S \\
 r \downarrow & & \downarrow s \\
 I \times I & \xrightarrow{u \times v} & J \times J
 \end{array}$$

S が symmetric $\Rightarrow r$ は symmetric
 S が transitive $\Rightarrow r$ は transitive
 $\hookrightarrow \tau \left(\begin{array}{c} \text{Per}(B) \\ \downarrow \\ B \end{array} \right)$ は fibration

S が reflexive
 $J \xrightarrow{g} S$
 $\delta_J \searrow \quad \downarrow s$
 $J \times J$

$$\Rightarrow \begin{array}{ccccc}
 & & g \circ u & & \\
 & & \curvearrowright & & \\
 I & \dashrightarrow & R & \rightarrow & S & (u \times u) \circ \delta_I \\
 \delta_I \searrow & & \downarrow r & & \downarrow s & = \delta_J \circ u \\
 & & I \times I & \xrightarrow{u \times u} & J \times J & = s \circ g \circ u
 \end{array}$$

$\hookrightarrow \tau$ は reflexive

$\nu \tau := \left(\begin{array}{c} \text{ERel}(B) \\ \downarrow \\ B \end{array} \right)$ は fibration

Exercise 1.3.10

$\text{Rel}'(\mathcal{B})$

objects $R \twoheadrightarrow I \times J$

morphism from $R \twoheadrightarrow I \times J$ to $S \twoheadrightarrow K \times L$ is

(u, v) such that $R \twoheadrightarrow S$ commutes.

$$\begin{array}{ccc} & \downarrow & \downarrow \\ & I \times J & \xrightarrow{u, v} & K \times L \end{array}$$

$$\begin{array}{ccc} \text{Rel}'(\mathcal{B}) & R \twoheadrightarrow I \times J & (u, v) \\ p \downarrow & \downarrow & \downarrow \\ \mathcal{B} \times \mathcal{B} & (I, J) & (u, v) \end{array}$$

p is fibration since monos are stable under pullback.

Exercise 1.3.11

\mathbb{E}
 $\downarrow p$
 \mathbb{B}

p is preordered \Leftarrow p is faithful

$$\begin{aligned} f, g &\in \mathbb{E}_I(X, Y) \\ \Rightarrow \text{id}_I &= p(f) = p(g) \in \mathbb{B}(pX, pY) = \mathbb{B}(I, I) \\ \Rightarrow &\text{— } p \text{ が "faithful な の で"—} \\ &f = g \end{aligned}$$

よって p は preordered

p is preordered \Rightarrow p is faithful

$$\begin{aligned} f, g &\in \mathbb{E}(X, Y) \wedge pf = pg \\ \Rightarrow f = g &\text{ が 自明に 言える の で } \\ &p \text{ は faithful} \end{aligned}$$

p is preordered \wedge f is above id_I とする

$$\begin{aligned} f \circ g &= f \circ h \\ \Rightarrow p(g) &= \text{id}_I \circ p(g) = p(f) \circ p(g) = p(f \circ g) = p(f \circ h) = p(h) \\ \Rightarrow g &= h \quad p \text{ は preordered と、よって faithful な の で} \\ \therefore f &\text{ は mono} \end{aligned}$$